Slotline Impedance

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Abstract —**A theoretical model is presented to compute the characteristic impedance and wavelength in a slotline printed on or embedded in a dielectric substrate. In this treatment the effects of fringing field caused by the finite width of the conducting strips are taken into account. The main task was to calculate the capacitance per unit length of the slotline. First, the Green's function for the potential of a pair of filament sources in a dielectric substrate is solved, which was used as a building block to construct the overall solution of the boundary value problem. Then the surface charge density on the conducting strips is found by using a moment method and imposing the source condition (equal potential) on the conductors. From the surface charge density, the characteristic impedance of the slotline is computed for various input parameters. The formulation is applicable to a single-sided, sandwiched, or double-sided slotline printed on or embedded in a dielectric substrate.**

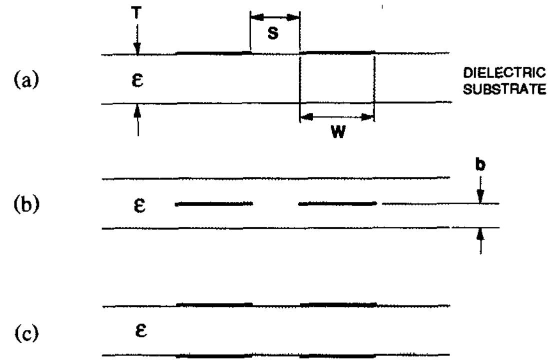

Fig. 1. Geometry of a general slotline printed on or embedded in a dielectric substrate. Note that the width of each conducting strip is finite. Formulation can be applied to a double-sided line also.

I. Introduction

A PRINTED slotline on a dielectric substrate without a ground plane is a novel transmission line suitable for application not only to microwave integrated circuits, but also to phased arrays as a feed line for tapered slot radiating elements. The impedance properties of a slotline have been studied by many researchers [1]–[12]. Some authors used conformal mapping to solve the boundary value problems, such as in [1]. This approach is somewhat limited in that not all geometries can be mapped into simple topologies for which standard solutions exist. For instance, when the slotline is embedded inside the substrate or when a double-sided slotline (one on each side of the substrate) is of interest, the mapping problem becomes intractable. Some authors have solved the E-field problem with the spectral-domain techniques [6]–[8] or variational methods [9], [10], both of which are mathematically involved. Others have offered approximate solutions for special cases of the slotline, such as high dielectric constants and small gap-to-thickness ratios [5], [7].

The purpose of this paper is to present an analytical solution which is conceptually simple and straightforward but general enough to be useful for a wide range of geometries of the slotline. In this method the slotline is considered to be infinitely long, and the conducting strips are infinitely thin. Furthermore, only the static potential field in the transverse plane of the transmission line is examined. The task is to compute the capacitance per unit length of the slotline by solving for the charge distribution on the conducting strips across the line. With this determined, the characteristic impedance can be computed by the standard method based on a quasi-TEM analysis [9]. Since the potential arising from a charged filament is well known, the boundary value problem can be simplified by dividing each conducting strip into a large number of filaments. To find the charge distribution on these subdivided strips, the well-known moment method [13] is employed and the source condition of equal potential on the perfect conductors is imposed. A computer program has been written to solve the linear equations by using the standard method of matrix inversion.

In the following sections the analytic formulation and several practical examples will be given. Numerical data for the slotline characteristic impedance for various input parameters will also be discussed. Comparisons with other studies indicated that the impedance results agree very well with the data published by Okean [1] and Yamashita [10].

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II. Formulation of the Problem

A conventional slotline is formed by etching a slot in a conducting plane printed on a dielectric substrate, as shown in Fig. 1(a). A more general case, however, is shown in Fig. 1(b), where the slotline is embedded in the substrate (it need not be sandwiched in the middle of the dielectric layer). Another case of interest is the double-sided line, shown in Fig. 1(c), where a slotline is symmetrically printed on each side of the substrate. For simplicity, it is assumed that the dielectric slab extends to infinity in the horizontal direction (x axis), but the width of each conducting strip, w, is finite.

The first step of this approach is to find the Green's function for the potential produced by two oppositely charged filaments embedded in the substrate, as shown in Fig. 2. Note that, for practical applications, only the odd-mode impedance between the filaments is studied. Let the filaments be symmetrically located at (x_0, y_0) and $(-x_0, y_0)$ inside the slab, which has a dielectric constant ϵ and a thickness T. The field point or the observation point is designated as (x, y).

Outside the substrate the potential field satisfies the source-free Laplace equation $\nabla^2\phi = 0$, so the homogeneous

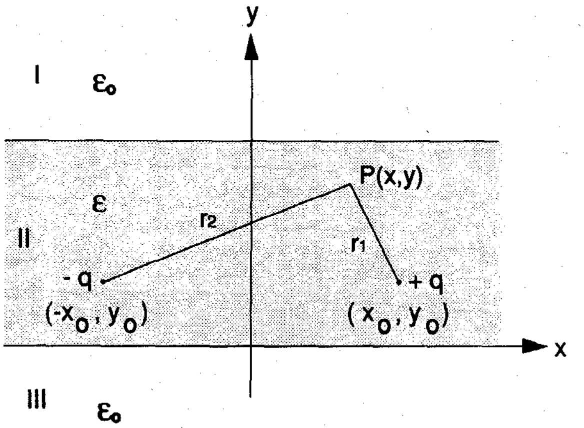

Fig. 2. Coordinates for the boundary value problem. The odd symmetry of the source filaments was taken into account to simplify the derivation.

solutions are in the forms of $\sin k_x x, \cos k_x x$, and $\exp(\pm k_x y)$. In terms of these Fourier components the general solution for region I above the substrate can be represented by

$$\phi(x,y)=\int_0^\infty A\sin k_x x\cdot e^{-k_x y}\,dk_x \tag{1}$$

$$k_x^2+k_y^2=0$$

in which A stands for the complex amplitude of the spectral component, a coefficient which remains to be found from the boundary conditions. Note that the cosine terms have been excluded from (1) because of the odd symmetry.

For region II, the solution can be written as

$$\phi(x,y)=\frac{q}{2\pi\epsilon}\ln\frac{r_2}{r_1}+\int_0^\infty \sin k_x x\left(B_1e^{k_x y}+B_2e^{-k_x y}\right)dk_x \tag{2}$$

where

$$r_1=\left[(x-x_0)^2+(y-y_0)^2\right]^{1/2}$$

$$r_2=\left[(x+x_0)^2+(y-y_0)^2\right]^{1/2}$$

are the distances from the field point to the two source points respectively. In the above, the first term represents the particular solution arising from the two line sources, and the second term represents all the homogeneous solutions required to account for the discontinuity at the dielectric interface.

Similarly for region III the solution is given by

$$\phi(x,y)=\int_0^\infty C\sin k_x x\cdot e^{k_x y}\,dk_x \tag{3}$$

The next step is to impose the boundary conditions so that the four coefficients A, B_1, B_2, and C can be determined. First, at $y=T$ the potential must be continuous, so from (1) and (2) it leads to

$$\int_0^\infty\left(Ae^{-k_xT}-B_1e^{k_xT}-B_2e^{-k_xT}\right)\sin k_x x\,dk_x = \frac{q}{2\pi\epsilon}\ln\frac{r_2}{r_1}\bigg|_{y=T}. \tag{4}$$

An inverse sine transform (for example, [14]) on (4) gives

$$Ae^{-k_xT}-B_1e^{k_xT}-B_2e^{-k_xT} = \frac{q}{2\pi^2\epsilon}\int_0^\infty \ln\frac{(x+x_0)^2+(T-y_0)^2}{(x-x_0)^2+(T-y_0)^2}\sin k_x x\,dx. \tag{5}$$

Using an integral table [15], one obtains

$$Ae^{-k_xT}-B_1e^{k_xT}-B_2e^{-k_xT} = \frac{q}{\pi\epsilon k_x}\sin k_x x_0 e^{-k_x(T-y_0)}. \tag{6}$$

The second boundary condition is to require the normal component of the electric displacement vector D (flux density) to be continuous across the dielectric boundary:

$$\epsilon_0\frac{\partial\phi}{\partial y}\bigg|_{y=T_+}=\epsilon\frac{\partial\phi}{\partial y}\bigg|_{y=T_-}$$

i.e.,

$$\int_0^\infty k_x\left[\epsilon\left(B_2e^{-k_xT}-B_1e^{k_xT}\right)-\epsilon_0Ae^{-k_xT}\right]\sin k_x x\,dk_x = \frac{q}{2\pi}\left[\frac{\partial}{\partial y}\ln\frac{r_2}{r_1}\right]_{y=T}. \tag{7}$$

This equation can be reduced by taking the derivative first and then carrying out the sine transform, or vice versa. The result is

$$\epsilon\left(B_2e^{-k_xT}-B_1e^{k_xT}\right)-\epsilon_0Ae^{-k_xT} = -\frac{q}{\pi k_x}\sin k_x x_0 e^{-k_x(T-y_0)}. \tag{8}$$

Following the same procedure one can impose the identical boundary conditions at $y=0$. This will provide two more equations for the four coefficients, with the results given by

$$B_1+B_2-C=-\frac{q}{\pi\epsilon}\frac{1}{k_x}e^{-k_xy_0}\sin k_x x_0 \tag{9}$$

and

$$\epsilon(B_1-B_2)-\epsilon_0C=-\frac{q}{\pi k_x}\sin k_x x_0 e^{-k_xy_0}. \tag{10}$$

From (6), (8), (9), and (10), one can solve for B_1 and B_2, giving

$$B_1=\frac{q\epsilon_-}{\pi\epsilon k_x}\sin k_x x_0\frac{\epsilon_+e^{-k_x(T-y_0)}+\epsilon_-e^{-k_x(T+y_0)}}{\epsilon_+^2e^{k_xT}-\epsilon_-^2e^{-k_xT}} \tag{11}$$

and

$$B_2=\frac{q\epsilon_-}{\pi\epsilon k_x}\sin k_x x_0\frac{\epsilon_+e^{k_x(T-y_0)}+\epsilon_-e^{-k_x(T-y_0)}}{\epsilon_+^2e^{k_xT}-\epsilon_-^2e^{-k_xT}} \tag{12}$$

where

$$\epsilon_+=\epsilon+\epsilon_0$$

$$\epsilon_-=\epsilon-\epsilon_0.$$

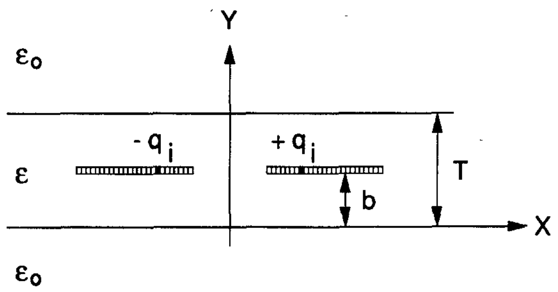

Fig. 3. Each conducting strip is represented by a large number of filaments. By linear superposition the potential produced by the two strips can be found from the Green's function.

When $\epsilon = \epsilon_0$, for the special case of no dielectric discontinuity, one can verify that $B_1 = B_2 = 0$, and

$$A = \frac{q}{\pi \epsilon_0 k_x} \sin k_x x_0 e^{-k_x y_0}$$

$$C = \frac{q}{\pi \epsilon_0 k_x} \sin k_x x_0 e^{k_x y_0}. \qquad (13)$$

In this case, as a check, the potential function in either region degenerates into

$$\phi(x,y) = \frac{q}{\pi\epsilon_0}\int_0^\infty \frac{1}{k_x} \sin k_x x_0 \sin k_x x e^{-k_x(y-y_0)} dk_x$$

$$= \frac{q}{4\pi\epsilon_0} \ln \frac{(x+x_0)^2 + (y-y_0)^2}{(x-x_0)^2+(y-y_0)^2}$$

$$= \frac{q}{2\pi\epsilon_0} \ln \frac{r_2}{r_1} \qquad (14)$$

which is nothing but the familiar potential function for two oppositely charged filaments in free space.

In summary, with B_1 and B_2 determined, the potential function given by (2) represents the Green's function of the electric field caused by a pair of delta filaments in region II. This Green's function will be used to construct the total potential arising from two conducting strips printed on or embedded in the substrate.

III. Moment Method for Charge Distribution

Referring to Fig. 3, let each conducting strip be divided into N segments, with each segment approximated by a filament. Assume that the ith filament on the right-hand strip, located at (x_i, b), carries charge density q_i per unit length and that its counterpart on the left-hand strip carries charge density $-q_i$. Then by linear superposition the total potential in the transverse plane arising from these N pairs of filaments is

$$\Phi(x,y) = \sum_{i=1}^{N} \phi_i(q_i, x, y, x_0 = x_i, y_0 = b). \qquad (15)$$

Now the task is to solve for the charge density distribution represented by N unknown q_i's. This is accomplished by imposing the equal potential condition on the N filaments, one at a time. This will provide N equations:

$$\Phi_j(x_j, y_j) = \sum_{i=1}^{N} \phi_{ij}(q_i, x_j, y_j = b, x_i) = V$$

$$(j = 1, 2, \cdots, N). \qquad (16)$$

In the above, when $j = i$, x_j is offset slightly from x_i to avoid singularity in the logarithmic term. More specifically, these N equations can be put in the form

$$q_1 P_{11} + q_2 P_{21} + \cdots + q_N P_{N1} = V$$
$$q_1 P_{12} + q_2 P_{22} + \cdots + q_N P_{N2} = V$$
$$\vdots$$
$$q_1 P_{1N} + q_2 P_{2N} + \cdots + q_N P_{NN} = V \qquad (17)$$

where P_{ij} is given by

$$P_{ij} = \frac{1}{2\pi\epsilon} \ln \frac{x_j + x_i}{|x_j - x_i|} + \frac{\epsilon_-}{\pi\epsilon}\int_0^\infty \frac{\sin k_x x_i \sin k_x x_j}{k_x\left(\epsilon_+^2 - \epsilon_-^2 e^{-2k_x T}\right)} \cdot \left[\epsilon_+ e^{-2k_x(T-y_0)} + 2\epsilon_- e^{-2k_x T} + \epsilon_+ e^{-2k_x y_0}\right] dk_x. \qquad (18)$$

A shorthand matrix notation for these equations is

$$[P_{ij}]\begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix} = \begin{pmatrix} V \\ \vdots \\ V \end{pmatrix}. \qquad (19)$$

From (19) the charge density is given by

$$\begin{pmatrix} q_1 \\ \vdots \\ q_N \end{pmatrix} = [P_{ij}]^{-1}\begin{pmatrix} V \\ \vdots \\ V \end{pmatrix} = [C_{ij}]\begin{pmatrix} V \\ \vdots \\ V \end{pmatrix} \qquad (20)$$

where C_{ij} are the elements of the inverse matrix of $[P_{ij}]$. The total charge Q is then

$$Q = \sum_{j=1}^{N} q_j = \sum_{j=1}^{N}\sum_{i=1}^{N} C_{ij} V. \qquad (21)$$

With Q determined, the unit capacitance of the transmission line is computed by

$$C' = \frac{Q}{2V} = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{N} C_{ij}. \qquad (22)$$

The factor 2 accounts for the fact that the two conducting strips are charged to plus and minus V volts with a difference of 2 V. From this capacitance the characteristic impedance, wavelength, and effective dielectric constant of the slotline can be computed by

$$Z_C = (C_0/C')^{1/2} Z_0$$
$$\lambda = (C_0/C')^{1/2} \lambda_0$$
$$\epsilon_{\text{eff}} = C'/C_0 \qquad (23)$$

where Z_0 and C_0 are the characteristic impedance and unit capacitance of the slotline when there is no dielectric loading of the substrate ($\epsilon = \epsilon_0$), and $Z_0 = (c_0 C_0)^{-1}$, with c_0 being the speed of light in free space.

It should be remarked that the parameter b determines the location of the conducting strips in or on the substrate. For $b = T$ the conducting strips are printed on top of the substrate. If b is less than T, the conducting strips are embedded inside the dielectric substrate. The formulation just derived is valid for either case. Furthermore, if it is a double-sided slotline, the potential function can be extended by linear superposition to include two parts, one for a slotline printed on the top ($b = T$), and the other on the bottom ($b = 0$).

The procedure described above is actually a moment method in its simplest form, i.e., a well-known point-matching technique by using delta functions as the basis and test functions. By this method many electrostatic and dynamic problems can be solved to a high degree of accuracy. A detailed treatment can be found in [13].

IV. Numerical Results

Based on the formulas presented in Section III, a computer program with fewer than 100 lines was written to compute the slotline impedance for various input parameters. For most practical designs, the dielectric constant ranges from 2 to 13, and the slot gap is usually small compared with the width of each conducting strip. Also, both the slot gap and thickness of the substrate are a small fraction of the wavelength.

It can be seen from (18) that, in theory, the contributions of all the spectral components must be integrated to match the boundary conditions. In practice, however, owing to the attenuating factors B_1 and B_2, the integration can be truncated at a finite point. It was found that a wavenumber of 100 for k_x is more than adequate to yield an accurate solution, and about 200 points are sufficient for the numerical integration. In the following paragraphs the characteristic impedance of the slotline as a function of various geometric parameters is discussed.

Shown in Fig. 4 is the characteristic impedance of a single-sided slotline versus the slot width for two different line widths of the conducting strips. The dielectric constant of the substrate is 9.6 and the thickness of the dielectric slab is 0.063 in. (1.6 mm). The impedance decreases as the width of the strip is increased. This effect indicates that the fringing field extends to the outer edge of the line, in this case, more than ten times the slot width.

In this figure the solid curves were computed by the theory presented here, while the dotted ones[1] were based on Okean's conformal mapping [1], which was known to agree with Yamashita's variational method [10]. It can be seen that the data derived from these approaches agree fairly well with each other. But the advantages of the new theory lie in the fact that it is straightforward and more flexible to use.

Fig. 5 illustrates the dependence of the characteristic impedance on the dielectric constant of the substrate. It was learned that a high dielectric constant and a slot width less than 20 mils (0.5 mm) are required to achieve a line impedance of 50 Ω. The parametric curves become more closely spaced when the dielectric constant is greater than 8. This implies that the effect of the dielectric constant is diminishing and the electric field does not penetrate too deeply below the gap in the substrate.

As the thickness of the dielectric substrate is increased from 0.02 to 0.04 in. (0.5–1.0 mm), the impedance is reduced by about 10%, as shown in Fig. 6. For a slot width of 40 mils (1 mm), the characteristic impedance stays as high as 80 Ω even for a relatively high K (9.6) material. So, in practice, it may be more convenient to include a transformer section in the feed line.

Again, the data of this example agree very well with that calculated by the method described in [1]. In fact, the difference is so small (less than 0.5%) that it cannot be shown in the plot. But the approximation used in [1] becomes inaccurate when the slot width or thickness is not small compared with the width of the conducting strip.

As defined in (23), an effective dielectric constant can be computed to determine the effect of dielectric loading and the wavelength along the slotline. An example is shown in Fig. 7, which corresponds to the case discussed in Fig. 5. For a slot width of 0.04 in., the effective dielectric constant is

[1]Courtesy of Kuan Lee's computer program, which is based on Okean's formula [1, fig. 1(d)].

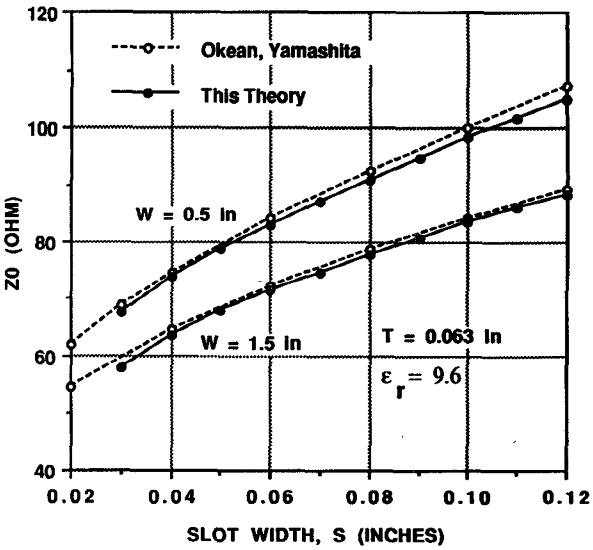

Fig. 4. Comparison between this theory and two published models. The dotted line was based on Okean's formula, which is known to agree with Yamashita's method.

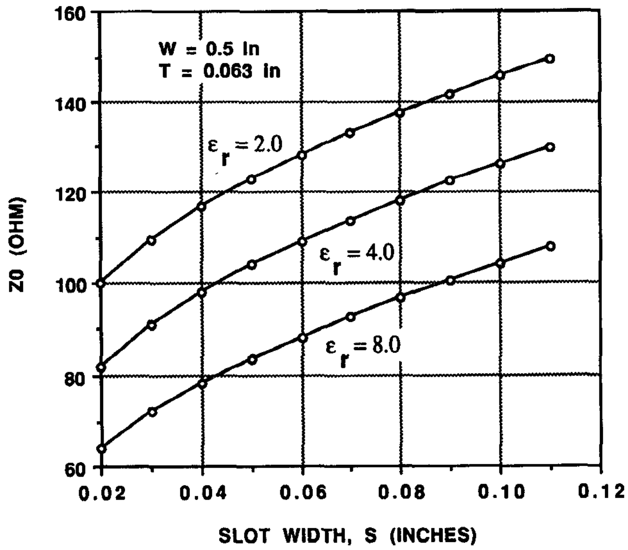

Fig. 5. Parametric curves of characteristic impedance versus slot width show the dependence of Z_0 on the dielectric constant of the substrate for the given width and thickness.

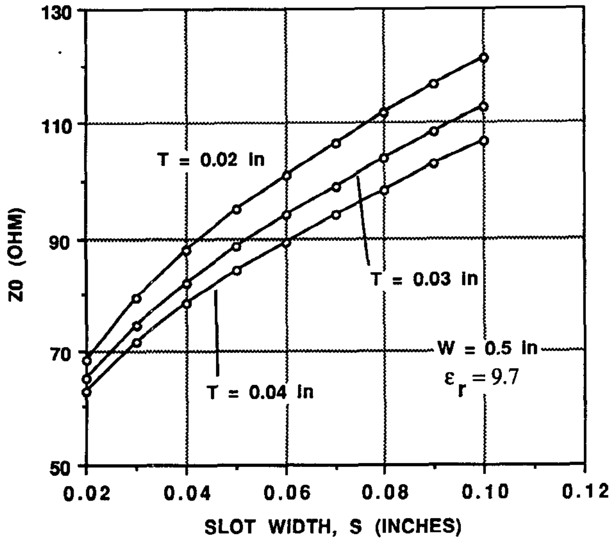

Fig. 6. Characteristic impedance versus slot width as a function of substrate thickness. This is for the case of $\epsilon_r = 9.7$ and $w = 0.5$ in.

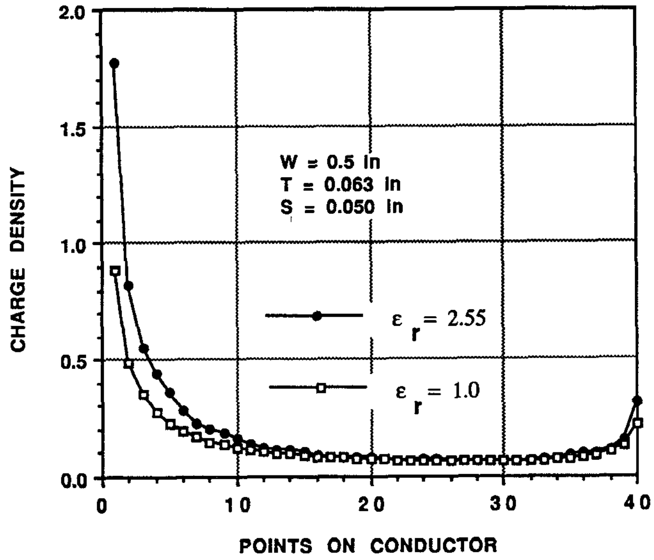

Fig. 8. Surface charge density on each conductor in the transverse plane. Note that the charge is shifted to the inner edge of the strip.

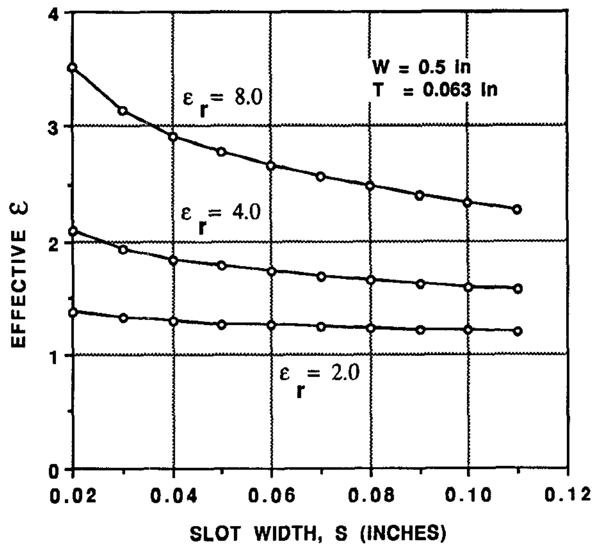

Fig. 7. Effective dielectric constant of the case shown in Fig. 5. For $s = 0.04$ in. the effective dielectric constant is approximately the square root of ϵ_r.

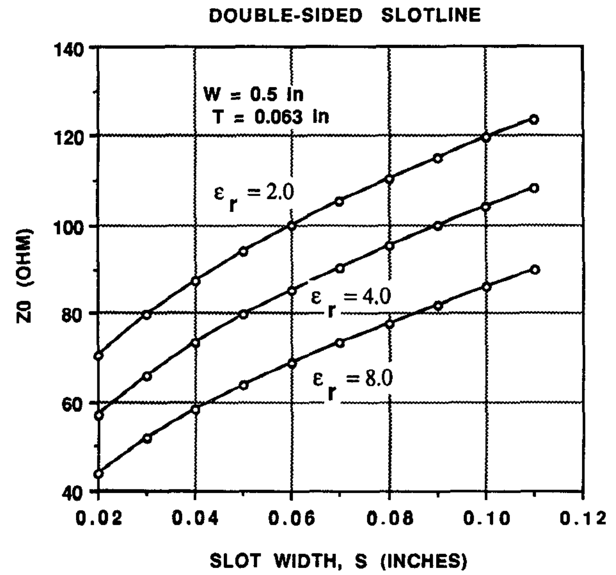

Fig. 9. Characteristic impedance of a double-sided slotline. It is about 25% lower than the value of a single-sided slotline.

approximately the square root of the actual dielectric constant of the substrate.

To gain some physical insight into the boundary value problem, it is interesting to study the electrostatic charge distribution across the conducting strip for a given geometry. Shown in Fig. 8 is the charge density profile on each conductor of the slotline for two low-dielectric cases. The horizontal axis displays the locations of 40 points on a conducting strip from the inner edge to the outer one. As expected, the charge density peaks at the inner edge of the slotline, but it is interesting to see a pedestal tail at the outside edge of the conducting strip. This is actually not too surprising if one remembers that when only a single strip is charged up the surface charge density symmetrically peaks at both outer edges owing to the repulsive force of the same kind of charge. When two parallel conducting strips are oppositely charged, however, the surface charge density on each strip is shifted to the inside edge, resulting in the density profile shown in Fig. 8.

The characteristic impedance of a double-sided slotline is shown in Fig. 9 for some typical design parameters. When compared with Fig. 5, the impedance is seen to drop about 25% because of the additional line printed on the other side of the substrate. A double-sided line is symmetrical in terms of dielectric loading and field distribution, so it is expected to be less dispersive and more suitable to be used as a feed line to drive a tapered slot element for wide-band array applications. Also, the symmetrical configuration may reduce the cross polarization in the radiation pattern of the tapered slot element.

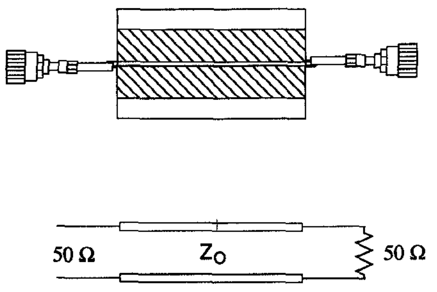

Fig. 10. An experimental setup to measure the characteristic impedance of the slotline. The inner conductor is soldered to one strip and the outer is soldered to the other. To balance the measurement, the connection is reversed on the other end of the slotline.

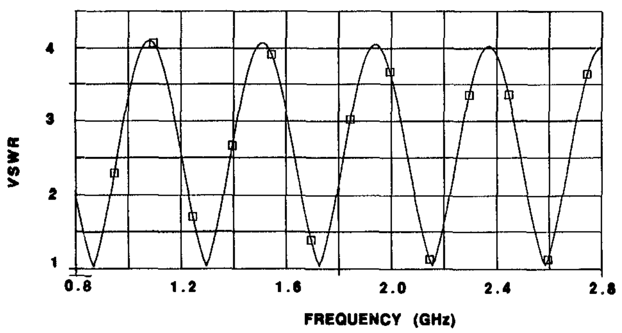

Fig. 11. A computed VSWR curve versus frequency for a 100 Ω line. From the peaks one can calculate the line impedance.

V. Measured Data

To verify the accuracy of the theoretical model, a simple experiment was conducted. As illustrated in Fig. 10, a slotline with 0.050 in. gap was printed on a low-K (2.55) substrate of 0.063 in. thickness. The slotline was terminated with a 50 Ω load, and its input VSWR was measured. It can be shown that for such an arrangement if the unknown characteristic impedance of the slotline is z_0, the maximum input VSWR of the slotline is given by[2]

$$\mathrm{VSWR}_m = (z_0/50)^2.$$

Thus the characteristic impedance of the slotline can be readily determined by taking the square root of the maximum VSWR and multiplying the number by 50 Ω. For example, the input VSWR of a 100 Ω line versus frequency is plotted in Fig. 11, in which the line length is 10 in. and the effective dielectric constant is 2.4.

In reality, a small amount of reactive loading at the input and output junctions is inevitable. In this case the peaks of the VSWR curve will vary depending on the local geometry of the junctions, the connectors, and the experimental setup. If the reactive loading is purely shunt capacitive, the peaks will gradually decay in amplitude. If, on the other hand, the loading is inductive, the peaks will grow. In general, some combination of capacitive and inductive loading exists and it is frequency dependent. So some error in the measurements was expected.

[2]See, for example, R. E. Collin's derivation in *Foundations for Microwave Engineering* (New York: McGraw-Hill, 1966, pp. 224–226). It becomes evident by setting $\Gamma_3 = -\Gamma_1$ in eq. (5.35) of this book to find the maximum reflection and VSWR for the case shown in Fig. 10.

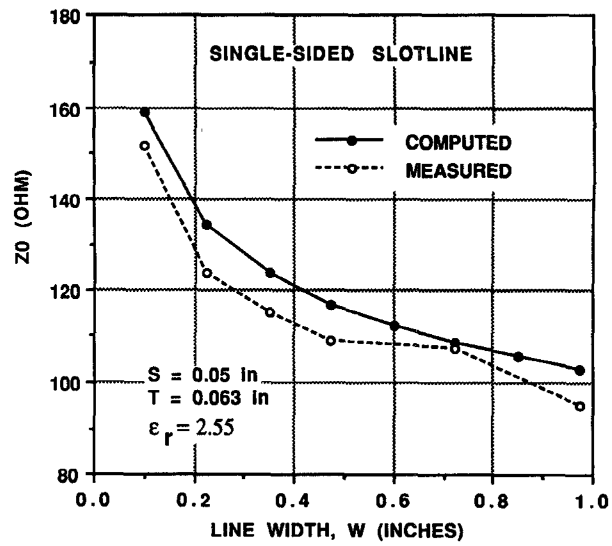

Fig. 12. Comparison between calculated and measured impedances for a single-sided slotline with a fixed gap. Measurements were conducted by trimming the line width of each strip.

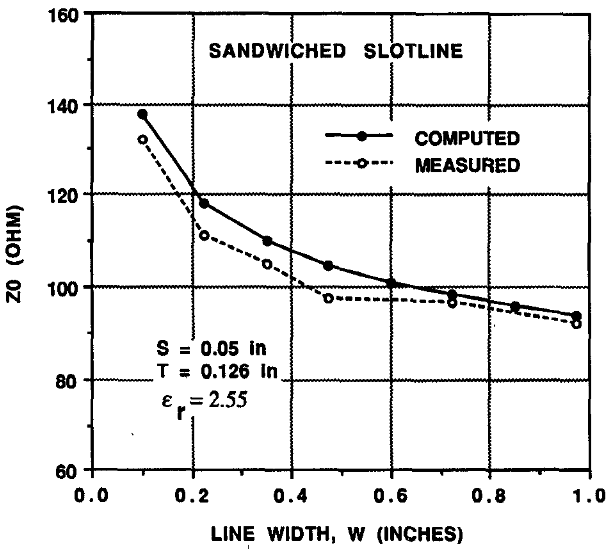

Fig. 13. Comparison between calculated and measured impedances for a sandwiched slotline.

Shown in Fig. 12 is a comparison of the computed and the measured characteristic impedance of a single-sided slotline just described. The data were taken by keeping the slot width constant while reducing the line width from 1.0 to 0.1 in. The variance between the computed and measured data is about 6% over a limited number of sample points. No rigorous effort has been made to resolve the discrepancy. However, the general trend of the impedance as a function of line width appears to be in good agreement.

As a further check, a sandwiched slotline was also constructed by placing a dielectric substrate of the same thickness on top of the previous slotline. Again, the characteristic impedance of the sandwiched line was measured. The impedance results versus the line width are shown in Fig. 13. It can be seen that in this case the match between the theory and the measured data is better than that of the single-sided line. At this time no investigation was made to determine if the numerical techniques or the measurements can be refined to produce a closer agreement.

VI. Conclusion

The boundary value problem of a slotline printed on or embedded in a dielectric substrate has been studied. The analysis was simplified by dividing each conducting strip into a large number of filaments, because the Green's function of a filament source in such a geometry can be readily derived. This Green's function was used as a building block to construct the overall solution for the potential generated by the slotline. To solve for the surface charge density on the conductors, a moment method was applied and the source condition of equal potential on the conducting strips was imposed. With the charge distribution determined, the capacitance per unit length of the slotline, and hence its characteristic impedance, could be computed for various input parameters.

The formulation presented here is general and valid for various slotline geometries. The solution can be used to compute the characteristic impedance and wavelength of a single-sided, a double-sided, or a sandwiched slotline. This model is conceptually simple and numerically efficient.

Acknowledgment

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